

FOUR YEARS B.Sc. (Honours) (CBCS) DEGREE EXAMINATION,
OCTOBER/NOVEMBER 2025.

THIRD SEMESTER

Mathematics

GROUP THEORY

(Regular)

Time : 3 Hours

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Max. Marks : 75

SECTION - A

Answer any FIVE of the following.

(Marks : 5×5 marks = 25 marks)

1. Show that a group G is abelian iff $(ab)^{-1} = a^{-1}b^{-1} \forall a, b \in G$.
2. Prove that all groups of order 4 and less are commutative.
3. If H is any subgroup of a group $(G, *)$ and $h \in G$. Then prove that $h \in G$ iff $hH = H = Hh$.
4. Prove that any two left (right) cosets of a subgroup are either identical or disjoint.
5. Show that every sub group of an abelian group is normal.
6. Prove that Intersection of any two normal subgroups of a group G is a normal subgroup.
7. If f is a homomorphism of a group G into a group G' then the Kernel of f is a normal sub group of G .
8. Find the regular permutations group isomorphic to the multiplicative group $\{1, -1, i, -i\}$.

SECTION - B

Answer FIVE of the following.

(Marks : 5×10 marks = 50 marks)

UNIT - I

9. (a) Prove that a finite semigroup G satisfying cancellation laws is a group.
Or
(b) If $*$ is defined on $\mathbb{Z}$, as $a * b = a + b + 1 \forall a, b \in \mathbb{Z}$ then show that $(\mathbb{Z}, *)$ is an abelian group

UNIT - II

10. (a) Prove that the union of two subgroups of a group G is a subgroup iff one is contained in another.
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Or
(b) State and prove Lagrange's theorem.

UNIT – III

11. (a) Prove that a subgroup H of a group G is a normal subgroup of G if and only if the product of two right (left) cosets of H in G is again a right (left) cosets of H in G .

Or

- (b) If M, N are two normal subgroup of G such that $M \cap N = \{e\}$ then show that every element of M commutes with every element of N .

UNIT – IV

12. (a) State and prove Fundamental theorem of Homomorphism of groups.

Or

- (b) If f is a homomorphism from a group G into a group G^1 then $\text{Ker } f$ is a normal subgroup of G .

UNIT – V

13. (a) If $f = (12345876)$, $g = (41587328)$ are cyclic permutation, then show that $(fg)^{-1} = g^{-1}f^{-1}$

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Or

- (b) State and prove Cayley's theorem.
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