

(MAT6SA)

(3110-6A)

B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2019.

(Regular)

Third Year — Sixth Semester

Part II — Mathematics

Paper VII (A) : (Elective A) — LAPLACE TRANSFORMS

Time : Three hours

Maximum : 75 marks

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE of the following questions.

1. Show that $L\{t^n\} = \frac{\sqrt{(n+1)}}{p^{n+1}}$.

$L\{t^n\} = \frac{\sqrt{(n+1)}}{p^{n+1}}$ అని చూపుము.

2. Evaluate $L\{3 \sin 2t - 5 \cos 2t\}$.

$L\{3 \sin 2t - 5 \cos 2t\}$ ని గణించుము.

3. If $L\{F(t)\} = f(s)$ then prove that $L\{e^{at} F(t)\} = f(s-a)$.

$L\{F(t)\} = f(s)$ అయితే $L\{e^{at} F(t)\} = f(s-a)$ అని చూపుము.

4. Evaluate $L\{F(t)\}$ if $F(t) = \begin{cases} (t-1)^2 & \text{if } t > 1 \\ 0 & \text{if } 0 < t < 1 \end{cases}$.

$F(t) = \begin{cases} (t-1)^2 & t > 1 \text{ అయినప్పుడు} \\ 0 & 0 < t < 1 \text{ అయితే} \end{cases}$ $L\{F(t)\}$ ని కనుగొనుము.

5. Evaluate $L\{\sin at - at \cos at\}$.

$L\{\sin at - at \cos at\}$ ని గణించుము.

6. Solve $L\left\{\frac{e^t - \cos t}{t}\right\}$.

$L\left\{\frac{e^t - \cos t}{t}\right\}$ ని సాధించుము.

7. Find $L^{-1}\left\{\frac{P}{(P+3)^{3/2}}\right\}$.
 $L^{-1}\left\{\frac{P}{(P+3)^{3/2}}\right\}$ ని కనుగొనుము.

8. Find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$.
 $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$ ని కనుగొనుము.

SECTION B — (5 × 10 = 50 marks)

Answer ALL the following questions.

9. (a) If $F(t)$ is a function of class A then prove that Laplace transform of $F(t)$ exists.
 $F(t)$ అనేది A - తరగతి ప్రమేయము అయితే లాప్లాస్ పరివర్తనం $F(t)$ కి వ్యవస్థితం అనిచూపుము.

Or

(b) Find $L\{\sin^2 at + (t^2 + 1)^2\}$.
 $L\{\sin^2 at + (t^2 + 1)^2\}$ కనుగొనుము.

10. (a) Given $L\left\{2\sqrt{\frac{t}{\pi}}\right\} = \frac{1}{s^{3/2}}$. Show that $\frac{1}{\sqrt{s}} = L\left\{\frac{1}{\sqrt{\pi t}}\right\}$.
 $L\left\{2\sqrt{\frac{t}{\pi}}\right\} = \frac{1}{s^{3/2}}$ అయిన $\frac{1}{\sqrt{s}} = L\left\{\frac{1}{\sqrt{\pi t}}\right\}$ అని చూపండి.

Or

(b) Find the Laplace transform of $F(t)$ where $F(t) = \begin{cases} \cos\left(t - \frac{2\pi}{3}\right) & \text{if } t > \frac{2\pi}{3} \\ 0 & \text{if } t < \frac{2\pi}{3} \end{cases}$

$F(t) = \begin{cases} \cos\left(t - \frac{2\pi}{3}\right) & \text{అయినప్పుడు } t > \frac{2\pi}{3} \\ 0 & \text{అయిన } t < \frac{2\pi}{3} \end{cases}$ $F(t)$ కి లాప్లాస్ పరివర్తనను కనుక్కోండి.

11. (a) Show that $L\{\sin \sqrt{t}\} = \frac{\sqrt{\pi} e^{-1/4s}}{2s^{3/2}}$.

$L\{\sin \sqrt{t}\} = \frac{\sqrt{\pi} e^{-1/4s}}{2s^{3/2}}$ అని చూపుము.

Or

(b) If $\text{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2} dx$ then prove that $L\{\text{erf}(\sqrt{t})\} = \frac{1}{s\sqrt{s+1}}$.

$\text{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2} dx$ అయితే $L\{\text{erf}(\sqrt{t})\} = \frac{1}{s\sqrt{s+1}}$ అని చూపుము.

12. (a) Evaluate $L^{-1}\left\{\frac{1}{(s-4)^5} + \frac{5}{(s-2)^2+5^2} + \frac{s+3}{(s+3)^2+6^2}\right\}$.

$L^{-1}\left\{\frac{1}{(s-4)^5} + \frac{5}{(s-2)^2+5^2} + \frac{s+3}{(s+3)^2+6^2}\right\}$ ను గణించుము.

Or

(b) Find $L^{-1}\left\{\frac{e^{-5p}}{(p-2)^4}\right\}$.

$L^{-1}\left\{\frac{e^{-5p}}{(p-2)^4}\right\}$ ని కనుగొనుము.

13. (a) If $L^{-1}\left\{\frac{e^{-\frac{1}{p}}}{\sqrt{p}}\right\} = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$ then find $L^{-1}\left\{\frac{e^{-\frac{a}{p}}}{\sqrt{p}}\right\}$.

$L^{-1}\left\{\frac{e^{-\frac{1}{p}}}{\sqrt{p}}\right\} = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$ అయితే $L^{-1}\left\{\frac{e^{-\frac{a}{p}}}{\sqrt{p}}\right\}$ ని కనుగొనుము.

Or

(b) Evaluate $L^{-1}\left\{\log\left(\frac{p+3}{p+2}\right)\right\}$.

$L^{-1}\left\{\log\left(\frac{p+3}{p+2}\right)\right\}$ ని కనుగొనుము.