(040910102)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

FIRST SEMESTER

Branch — Mathematics

Paper I — ALGEBRA

(Old Syllabus)

e : 3 Hours *Max. Marks : 80*

PART – A

Answer any FIVE questions. Each question carries 4 marks.

(Marks : 5 × 4 marks = 20 marks)

State and prove Cayley's theorem.

Prove that there are no simple group b order 63.

Let $f : R \to S$ be a homomorphic of a ring R into a ring S. Then prove that $\ker f = (0)$ if and only if f is 1–1.

Let R be a Boolean ring. Then show that each prime ideal $P \neq R$ is maximal.

Prove that an irreducible element in a commutative principal ideal domain (PID) is always prime.

Prove that every euclidean domain is a PID.

If R is a ring with unity then prove that an R-module M is cyclic if and only if $M \simeq R/I$ for some left ideal I of R.

State and prove Schur's lemma.

PART – B

Answer ONE question from each Unit.

(Marks : 4 × 15 marks = 60 marks)

UNIT – I

(a) State and prove Burnside theorem.

(b) Prove that every group of order p^2 (p be prime) is abelian.

Or

State and prove second and third Sylow theorems.

[P.T.O]

UNIT - II

11. (a) State and prove fundamental theorem of homomorphisms.

    (b) Let R be a ring. Then prove that $(R_n)^{op} \cong (R^{op})_n$.

Or

12. (a) In a non zero commutative ring with unity, prove that an ideal M is maximal if and only if R/M is a field.

    (b) Define nil potent. Prove that if R is a non zero ring unity 1 and I is an ideal of R such that $I \neq R$, then there exists a maximal ideal M of such that $I \subseteq M$.

UNIT - III

13. Prove that every PID is a UFD, but a UFD is not necessarily a PID.

Or

14. Let R be a unique factorization domain. Then prove that the polynomial ring $R[x]$ over R is also a unique factorization domain.

UNIT - IV

15. Let R be a ring with unity and let M be an R-module. Then prove that the following statements are equivalent :

    (a) M is simple.

    (b) $M \neq (0)$ and M is generated by any $x(\neq 0) \in M$.

    (c) $M \cong R/I$, where I is a maximal left ideal of R.

Or

16. Let M be a finitely generated free module over a commutative ring R. Then prove that all basis of M have the same number of elements.
