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M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

THIRD SEMESTER

Branch — Mathematics/Applied Mathematics

Paper IV — NUMBER THEORY

(New Syllabus)

Time : 3 Hours *Max. Marks : 80*

PART - A

Answer any FOUR questions. Each question carries 5 marks.

(Marks : 4 × 5 marks = 20 marks)

1. If $n \geq 1$, prove that $\sum_{d/n} \phi(d) = n$.

2. If $n \geq 1$, prove that $\log n = \sum_{d/n} \wedge(d)$.

3. If $x \geq 1$, prove that $\sum_{n \leq x} \frac{1}{n} = \log x + C + O\left(\frac{1}{x}\right)$.

4. For $x > 1$, prove that $\sum_{n \leq x} \phi(n) = \frac{3}{\pi^2} x^2 + O(x \log x)$.

5. Show that congruence is an equivalence relation.

6. If $ac \equiv bc \pmod m$ and if $d = (m, c)$, then prove that $a \equiv b\left(\text{mod}\, \frac{m}{d}\right)$.

7. For every odd prime p, prove that

$$(-1/p) = (-1)^{(p-1)/2} = \begin{cases} 1 & if \quad p \equiv 1 \pmod 4 \\ -1 & if \quad p \equiv 3 \pmod 4 \end{cases}.$$

8. Determine the odd primes p for which 3 is quartatic residue and those for which it is a non residue.

PART - B

Answer ONE questions from each Unit. Each question carries 15 marks.

(Marks : 4 × 15 marks = 60 marks)

UNIT - I

9. (a) Show that Dirichlet multiplication is commutative and associative.

   (b) For all f, prove that $I * f = f * I = f$

Or

10. (a) Let f be multiplicative. Then prove that f is completely multiplicative if and only $f^{-1}(n) = \mu(n) f(n)$ for all $n \geq 1$.

(b) If f and g are arithmetical functions, then prove that $(f * g)^{1} = f' * g + f * g'$

UNIT – II

11. for all $x \geq 1$, prove that $\sum d(n) = x \log x + (2C-1)x + O(\sqrt{x})$ where C is Euler's constant.

Or

12. for $x \geq 2$, prove that $\sum_{p \leq x} \left[\frac{x}{p}\right] \log p = x \log x + O(x)$, where the sum is extended over all prim $\leq x$.

UNIT – III

13. (a) Assume $(a, m) = 1$. then prove that the linear congruence $ax \equiv b \pmod{m}$ has exact one solution.

(b) If a prime p does not divide a, then prove that $a^{p-1} \equiv 1 \pmod{p}$

Or

14. (a) State and prove Lagrange's theorem.

(b) State and prove Wilson's theorem.

UNIT – IV

15. State and prove Gauss' lemma.

Or

16. (a) State and process Reciprocity law for Jacobi symbols.

(b) Let x be an odd integer. If $\alpha \geq 3$, prove that, prove that $x^{\phi(2^{\alpha})/2} \equiv 1 \pmod{2^{\alpha}}$.
