(040930302)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017

THIRD SEMESTER

Branch – Mathematics / Applied Mathematics

Paper III — DIFFERENTIAL GEOMETRY

(New Regulation)

Time : 3 Hours *Max. Marks : 80*

PART – A

Answer any FOUR of the following questions.

Each question carries 5 marks.

(Marks : 4 × 5 marks = 20 marks)

1. Find the length of the curve given as the intersection of the surfaces $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,\ x=a\cos(z/a)$, from the point (a, o, o) to the point (x, y, z).

2. Show that the involutes of a circular helix are plane curves.

3. Find the surface of revolution which is isometric with a region of the right helicoid.

4. Prove that the curves of the family $v^3/u^3=$ constant are geodesics on a surface with metric $v^2du^2-2uvdudv+2u^2dv^2(u>0, v>0)$.

5. State and prove Liouville's formula for Kg.

6. Show that every point on a surface has a neighbourhood which can be mapped conformally on a region of the plane.

7. Write about Rodrigue's formula.

8. Show that asymptotic lines are self conjugate.

PART – B

Answer ONE question from each Unit. Each question carries 15 marks.

(Marks : 4 × 15 marks = 60 marks)

UNIT – I

9. Obtain the curvature and torsion of the curve of intersection of the two quadratic surfaces $ax^2+by^2+cz^2=1,\ a^1x^2+b^1y^2+c^1z^2=1$.

Or

10. (a) If a curve lies on a sphere show that p and σ are related by $\frac{d}{ds}(\sigma\rho^1)+\frac{\rho}{\sigma}=0$.

(b) Show that the torsion of an involute of a curve is equal to $\frac{\rho(\sigma\rho^1-\sigma^1\rho)}{(\rho^2+\sigma^2)(c-S)}$.

UNIT – II

11. Find the coefficients of the direction which makes an angle $\frac{\pi}{2}$ with the direction whose coefficients are (l, m).

Or

12. (a) On the paraboliod $x^2 - y^2 = Z$, find the orthogonal trajectories of the sections by the planes Z= constant.

(b) A helicoid is generated by the screw motion of a straight line which meets the axis at an angle α. Find the orthogonal trajectories of the generators.

UNIT – III

13. State and prove the necessary and sufficient condition for the curve V=C be a geodesic on the general surface then H^2/E is independent of u.

Or

14. State and prove Tissot's theorem.

UNIT – IV

15. State and prove Mainardi-Codazzi equations.

Or

16. Prove that a necessary and sufficient condition that a curve on a surface be a line of curvature is that the surface normals along the curve from a developable.
