(040930102)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

THIRD SEMESTER

Branch – Mathematics

Paper I — COMMUTATIVE ALGEBRA

(New Regulation)

Time : 3 Hours

Max. Marks : 80

PART – A

Answer any FOUR questions.

Each question carries 5 marks.

(Marks : 4 × 5 marks = 20 marks)

1. If N and L are submodules of an R-module M. Then prove that $(L + N) - N \simeq L - (L \cap N)$.
2. Prove that Radical of an Ideal A is an Ideal containing A.
3. If M has finite length and N is a submodule of M, then prove that composition differences of M are the composition differences of N and M–N together.
4. If M is completely reducible, then prove that every submodule of M is completely reducible.
5. Prove that every non-unit a in a Noetherian domain R is a product of irreducible elements.
6. Prove that a ring with identity satisfying the d.c.c. has only a finite number of prime ideals $M_1, M_2, \ldots M_n$ all of them maximal.
7. Let R be a ring and A be an ideal of R admitting an irredundant primary representation $A = \bigcap_i Q_i$. Suppose all the Q_i are prime ideals then prove that A to be its radical.
8. Prove that the set of Nilpotent elements in a Noetherian ring R is the intersection of Isolated prime ideals of zero ideal.

PART – B

Answer ONE question from each Unit.

Each question carries 15 marks.

(Marks : 4 × 15 marks = 60 marks)

UNIT – I

9. Let U be an ideal different from R then prove that
   (a) U is prime if and only if R/U has no zero divisors.
   (b) If R has an identity, then U is maximal if and only if R/U is a field.

Or

10. Let Q be a primary ideal in a ring R. If P is the radical of Q then prove that P is a prime ideal, moreover if $ab \in Q$ and $a \notin Q$, then prove that $b \in P$.

[P.T.O]

UNIT – II

11. State and prove Jordan's theorem.

Or

12. (a) If a module M has a composition series then prove that any two composition series are equivalent.

(b) Prove that an R-module M satisfying the d.c.c. is a direct sum of finite number of indecomposable submodules.

UNIT – III

13. Let R be a ring with Identity. For R to satisfy the d.c.c. prove that it is necessary and sufficient that it satisfies the a.c.c. and every prime ideal of R different from R be maximal.

Or

14. (a) Prove that in a ring with a.c.c, every ideal is a finite intersection of irreducible ideals.

(b) Prove that in a ring with a.c.c, every irreducible ideal is primary.

UNIT – IV

15. Let R be a Noetherian ring, A and B be two ideals of R such that $A \neq R$. Then prove that $A = A : B$ if and only if B is contained in no prime ideal of A.

Or

16. (a) State and prove Krull's intersection theorem.

(b) Let R be a ring and M an ideal of R. If $\{A_x\}$, a family of ideals of R which are closed with respect to M, then prove $\bigcap_x A_x$ is closed.
