(205MAT17/205APM17)

M.Sc. DEGREE EXAMINATION, APRIL 2018

SECOND SEMESTER

Branch — Mathematics / Applied Mathematics

ADVANCED COMPLEX ANALYSIS

(New Syllabus for Batch 2017)

Time : 3 Hours *Max. Marks : 80*

SECTION – A

Answer any FOUR of the following questions.

(Marks : 4 × 5 marks = 20 marks)

1. Find the Laurent expansion of $f(z) = \frac{1}{z(1-z)}$ in the annulus $0 < |z-1| < 1$.

2. Find the singular points and investigate the behaviour at infinity for the function $f(z) = \frac{1}{z - z^3}$.

3. State and prove Rouche's theorem.

4. Find the number of zeros of the polynomial $z^7 - 5z^4 + z^2 - 2$ inside the unit circle $|z| = 1$.

5. Find the analytic function $f(z)$ whose real part is $u(x,y) = x^2 - y^2 + 2$.

6. State and prove Riemann's mapping theorem.

7. If an entire function $f(z)$ has no zeros then prove that $f(z)$ is of the form $f(z) = e^{g(z)}$, where $g(z)$ is an entire function.

8. Define infinite product, prove that $\prod_{n=2}^{\infty}\left(1 - \frac{1}{n^2}\right) = \frac{1}{2}$.

SECTION – B

Answer FOUR of the following questions choosing One from each Unit.

(Marks : 4 × 15 marks = 60 marks)

UNIT – I

9. (a) State and prove Laurent's theorem.

Or

(b) Let $f(z)$ be a single-valued analytic function defined on the domain $D : 0 < |z - z_0| < R$. Then prove that z_0 is a regular point of $f(z)$ if and only if $f(z)$ is bounded in some deleted neighborhood of z_0.

[P.T.O]

UNIT – II

10. (a) State and prove Residue theorem.

Or

(b) (i) State and prove Maximum modulus principle.

(ii) State and prove Schwarz's lemma.

UNIT – III

11. (a) State and prove Poisson's integral formula and deduce Schwarz's formula.

Or

(b) State and prove Dirichlet problem.

UNIT – IV

12. (a) State and prove Weierstrass theorem.

Or

(b) State and prove Mittag-Leffler's theorem.