(202MAT17/202APM17)

M.Sc. DEGREE EXAMINATION, APRIL 2018.

SECOND SEMESTER

Branch — Mathematics/Applied Mathematics

MEASURE AND INTEGRATION

(New Syllabus for batch 2017)

Time : 3 Hours *Max. Marks : 80*

SECTION – A

Answer any FOUR of the following questions.

(Marks : 4 × 5 marks = 20 marks)

1. If E_1 and E_2 are measurable sets, then show that $E_1 \cup E_2$ is measurable.

2. Prove that every Borel set is measurable.

3. If ϕ and ψ are simple functions which vanish outside a set of finite measure then prove that $\int(a\phi + b\psi) = a\int\phi + b\int\psi$. Also prove that $\int\phi \geq \int\psi$ if $\phi \geq \psi$ almost everywhere.

4. State and prove Fatou's lemma.

5. If F is of bounded variation on $[a, b]$ then prove that $T_a^b = P_a^b + N_a^b$ and $f(b) - f(a) = P_a^b - N_a^b$.

6. State and prove Jeuseu inequality.

7. If $1 \leq p \leq \infty$ and a, b, t are nonnegative then prove that $(a+tb)^P \geq a^P + ptba^{P-1}$.

8. Show that $\|f+g\|_\infty \leq \|f\|_\infty + \|g\|_\infty$.

SECTION – B

Answer FOUR of the following questions choosing ONE from each Unit.

(Marks : 4 × 15 marks = 60 marks)

UNIT I

9. (a) Prove that the outer measure of an interval is its length.

Or

(b) (i) Prove that the interval (a, ∞) is measurable.

(ii) Let $< E_n >$ be an infinite decreasing sequence of measurable sets, that is a sequence with $E_{u+1} < E_u$ for each n. Let mE_1 be finite. Then prove that $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} mE_n$.

UNIT II

10. (a) Let f be defined and bounded on a measurable set E with in E finite, In order th $\inf_{f \le \psi} \int_E \psi(x)dx = Sup_{f \ge \phi} \int_E \phi(x)dx$ for all simple functions ϕ and ψ, prove that it is necess and sufficient that f is measurable.

Or

(b) (i) State and prove Bounded Convergence Theorem.

(ii) State and prove Monotone Convergence Theorem.

UNIT III

11. (a) Let f be an increasing real valued function on the interval $[a, b]$. Then prove tha differentiable almost everywhere. Also prove that the derivative f' is measurable $\int_a^b f'(x)dx \le f(b) - f(a)$.

Or

(b) (i) If f is bounded and measurable on $[a, b]$ and $F(x) = \int_b^x f(t)dt + F(a)$ then prove $F'(x) = f(x)$ for almost all x in $[a, b]$.

(ii) Let f be an intergrable function on $[a, b]$ and suppose that $F(x) = F(a) + \int_a^x f$

Then prove that $F'(x) = f(x)$ for almost all x in $[a, b]$.

UNIT IV

12. (a) Prove that the L^p spaces are complete.

Or

(b) State and prove Riesz Representation Theorem.
