

(201MAT17/201APM17)

M.Sc. DEGREE EXAMINATION, APRIL 2018

SECOND SEMESTER

Branch — Mathematics / Applied Mathematics

DISCRETE MATHEMATICS

(New Syllabus for batch 2017)

Time : 3 Hours

Max. Marks : 80

SECTION - A

Answer any FOUR of the following questions.

(Marks :  $4 \times 5$  marks = 20 marks)

1. Obtain disjunctive normal form of  $P \wedge (P \rightarrow Q)$ .
2. Describe Rules of inference.
3. Symbolize the expression "All men are giants".
4. Negate the statement "Ottawa is a small town".
5. Prove that every chain is a Distributive Lattice.
6. Obtain the sum-of-products canonical form of the Boolean expression  $x_1 \oplus (x_2 * x_3^1)$ .
7. If  $S_i \equiv S_j$ , then prove that for any input sequence  $x, \delta(s_i, x) \equiv \delta(s_j, x)$ .
8. Define a Graph. Give an example.

SECTION - B

Answer FOUR of the following questions choosing One from each Unit.

(Marks :  $4 \times 15$  marks = 60 marks)

UNIT - I

9. (a) (i) Show that  $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$  is a tautology.
- (ii) Obtain the principal conjunctive normal form of  $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$ .

Or

- (b) (i) Prove that  $R \rightarrow S$  is derived from the premises  $P \rightarrow (Q \rightarrow S)$ ,  $\neg R \vee P$  and  $Q$ .
- (ii) Show that  $P \wedge \neg P \wedge Q \Rightarrow R$ , by using automatic theorem proving.

[P.T.O]

## UNIT - II

10. (a) Indicate the variables that are free and bound and also show the scope of the quantifier in  $(\exists x)(P(x) \wedge R(x)) \rightarrow (\exists x)P(x) \wedge Q(x)$ .

Or

- (b) (i) Explain the theory of inference for the predicate calculus.  
(ii) Prove that  $(\exists x)(P(x) \rightarrow Q(x)) \wedge (\exists x)(Q(x) \rightarrow R(x)) \Rightarrow (\exists x)(P(x) \rightarrow R(x))$ .

## UNIT - III

11. (a) Let  $\langle L, \leq \rangle$  be a lattice in which  $*$  and  $\oplus$  denote the operations of meet and join respectively. Then prove that for any  $a, b \in L$ ,  $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$ .

Or

- (b) (i) Obtain the product-of-sums canonical form of the expression  $[(x_1 + x_2)(x_3 x_4)']$ .  
(ii) Use the Karnaugh map representation to find a minimal sum-of-products expression of the function  $f(a, b, c) = \sum(0, 1, 2, 3, 13, 15)$ .

## UNIT - IV

12. (a) Let  $s$  be any state in a finite-state machine and  $x$  and  $y$  be any words. Then prove that  $\delta(s, xy) = \delta(\delta(s, x), y)$  and  $\lambda(s, xy) = \lambda(\delta(s, x), y)$ .

Or

- (b) For any  $n \times n$  Boolean matrix  $A$ , show that  $(I + A)^{(2)} = (I + A) \wedge (I + A) = I + A + A$  where  $I$  is the  $n \times n$  identity matrix and  $A^{(2)} = A \wedge A$ . Also show that for any positive integer  $r$ ,  $(I + A)^{(r)} = I + A + A^{(2)} + \dots + A^{(r)}$ .