(040920402/24941B)

M.Sc. DEGREE EXAMINATION, APRIL 2018

SECOND SEMESTER

Branch — Applied Mathematics/Mathematics

TOPOLOGY

(Old Syllabus for Batch 2013 to 2016)

Time : 3 Hours *Max. Marks : 80*

PART – A

Answer any FIVE of the following.

(Marks : 5 × 4 marks = 20 marks)

1. Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous at x_0 if and only if $x_n \to x_0 \Rightarrow f(x_n) \to f(x_0)$.

2. State and prove Minkowski's inequality.

3. Let X be a topological space and A a subset of X. Then show that $\overline{A} = A \cup D(A)$ where $D(A)$ is a derived set of A.

4. Let X be a second countable space. Then prove that any open base for X has a countable subclass which is also an open base.

5. Show that any continuous image of a compact space is compact.

6. Prove that a topological space is compact if every basic open cover has a finite subcover.

7. Show that the product of any non-empty class of Hausdorff spaces is a Hausdorff space.

8. Show that any continuous image of a connected space is connected.

PART – B

Answer ONE question from each Unit.

(Marks : 4 × 15 marks = 60 marks)

UNIT – I

9. State and prove Cantor's Intersection theorem.

Or

10. Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.

[P.T.O]

UNIT – II

11. (a) State and prove Lindelof's theorem.

    (b) Show that every separable metric space is second countable.

Or

12. Let X be any non-empty set and let S be an arbitrary class of subsets of X. Then prove tha can serve as an open subbase for a topology on X, in the sense that the class of all unions finite intersections of sets in S is a topology.

UNIT – III

13. (a) Show that a metric space is compact if and only if it is complete and totally bounded.

    (b) Prove that a closed subspace of a complete metric space is compact if and only if it totally bounded.

Or

14. State and prove Ascoli's theorem.

UNIT – IV

15. (a) State and prove Urysohn's lemma.

    (b) Show that the spaces R^n and C^n are connected.

Or

16. State and prove Tietze extension theorem.

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