(040920202/245921B)

M.Sc. DEGREE EXAMINATION, APRIL 2018

SECOND SEMESTER

Branch – Applied Mathematics/Mathematics

MEASURE AND INTEGRATION

(Old Syllabus for batch 2013 to 2016)

Time : 3 Hours *Max. Marks : 80*

PART – A

Answer any FIVE of the following.

(Marks : 5 × 4 marks = 20 marks)

1. Show that every subset of a countable set is countable.

2. State and prove Lindelof theorems.

3. Let $\{A_n\}$ be a countable collection of sets of real numbers. Then prove that $m^*\left(\bigcup A_n\right) \leq \sum m^*(A_n)$.

4. If $m^*E = 0$ then show that E is measurable.

5. Let f be a bounded function defined on $[a,b]$. If f is Riemann integrable on $[a,b]$ then prove that f is measurable and

$$R\int_a^b f(x)\,dx = \int_a^b f(x)\,dx.$$

6. State and prove Ratou's Lemma.

7. Prove that a function f is of bounded variation on $[a,b]$ if and only if f is the difference of two monotone real valued functions on $[a,b]$.

8. If f is integrable on $[a,b]$, then show that the function F defined by

$$F(x) = \int_a^x f(t)\,dt$$

is a continuous function of bounded variation on $[a,b]$.

[P.T.O]

PART – B

Answer ONE question from each Unit.

(Marks : 4 × 15 marks = 60 marks)

UNIT – I

9. State and prove Heine-Borel theorem.

Or

10. (a) If f is a real valued function defined and continuous on a closed and bounded set of real numbers, prove that it is uniformly continuous.

(b) Let f be a real valued function defined on $(-\infty,\infty)$. Then show that f is continuous if and only if for each open set O of real numbers $f^{-1}[0]$ is an open set.

UNIT – II

11. Show that the outer measure of an interval is its length.

Or

12. Let $\{E_n\}$ be an infinite decreasing sequence of measurable sets and let mE_1 be finite. Then show that $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \underset{n\to\infty}{Lt}\, mE_n$.

UNIT – III

13. Let f be defined and bounded on a measurable set E with mE finite. Prove that necessary and sufficient condition for f is measurable is

$\inf_{f\le\psi} \int_E \psi(x)\,dx = \inf_{f\le\phi} \int_E \phi'(x)\,dx$ for all simple functions ϕ and ψ.

Or

14. (a) State and prove bounded convergence theorems.

(b) Let f be a measurable function which is integrable over a set E. Then show that given $\epsilon > 0$ there is a $\delta > 0$ such that for every set $A \subset E$ with $mA < \delta$, we have $\int_A f < \epsilon$.

UNIT – IV

15. State and prove Vitali covering lemma.

Or

16. If f is absolutely continuous on $[a,b]$ and $f'(x) = 0$ a.e. then show that f is a constant.
