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M.Sc. DEGREE EXAMINATION, APRIL 2018.

FOURTH SEMESTER

Branch — Mathematics

Paper IV — BANACH ALGEBRAS

Time : 3 Hours *Max. Marks : 80*

PART – A

Answer any FIVE of the following.

(Marks : 5 × 4 marks = 20 marks)

1. If A is a division algebra then prove that it is equal to the set of all scalar multiples of the identity.

2. If r is an element of A with property that $1-xr$ is regular for every x then prove that r is in R.

3. If A is a commutative Banach algebra, prove that $r(x)=\|x\| \Rightarrow \|\hat{x}\|=\|x\|$ for every x.

4. If x is a normal element in a B^*–algebra, then prove that $\|x^2\|=\|x\|^2$.

5. If X is a compact Hausdorff space, then prove that every closed ideal in $\mathcal{C}(X)$ is the intersection of the maximal ideals which contain it.

6. Let X be a compact Hausdorff space and A be a Banach sub algebra of $\mathcal{C}(X)$ If f is a real function in A which is regular in $\mathcal{C}(X)$ then prove that it is also regular in A.

7. Prove that every convex compact subspace of a Banach space is a fixed point space.

8. If x is a non zero element in a Boolean ring R then prove that there exists a homeomorphism h of R onto $\{0,1\}$ such that $h(x)=1$.

PART – B

Answer ONE question from each Unit.

(Marks : 4 × 15 marks = 60 marks)

UNIT – I

9. (a) Prove that the boundary of S is a subset of Z.

   (b) Show that $\sigma(x)$ is non empty.

Or

10. (a) Prove that $r(x) = \lim \left\|x^n\right\|^{1/n}$.

    (b) Prove that A/R is a semi–simple Banach algebra.

UNIT – II

11. (a) Prove that $r \to fr$ is a one-to-one mapping of the set m of all maximal ideals in the set of all its multiplicative functionals.

(b) If A is self-adjoint then prove that $\hat{A}$ is dense in $e(m)$.

Or

12. State and prove Gelfand – Neumark theorem.

UNIT – III

13. Let A be a commutative C^* – algebra of operators on H. If an operator T in A is reg $B(H)$ then prove that it is also regular in A and therefore the spectrum of T as an o on H equals its spectrum as an element of A.

Or

14. Let N be a normal operator on H, A the commutative C^* – algebra generated by N the space of maximal ideals in A. Then prove that the function $\hat{N}$ in $e(m$ corresponds to N under the Gelfand mapping is a homeomorphism, of m onto $\sigma(N)$.

UNIT–IV

15. State and prove Picard's theorem.

Or

16. State and prove stone representation theorem.

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