(203MAT17/203APM17)

M.Sc. DEGREE EXAMINATION, APRIL 2018

SECOND SEMESTER

Branch — Mathematics/Applied Mathematics

Paper — PARTIAL DIFFERENTIAL EQUATIONS

(New syllabus for batch 2017)

Time : 3 Hours *Max. Marks : 80*

SECTION – A

Answer any FOUR of the following questions.

(Marks : 4 × 5 marks = 20 marks)

1. Find the orthogonal trajectories on the cone $x^2+y^2=z^2\tan^2\alpha$ of its intersections, with the family of planes parallel to $z=0$.

2. Verify that the differential equation $(y^2+yz)dx+(xz+z^2)\,dy+(y^2-xy)dz=0$ is integrable and find its primitive.

3. Eliminate the arbitrary function f from the equation $z=f(x-y)$.

4. Find the complete integral of the equation $(p^2+q^2)y=qz$.

5. If $z=f(x^2-y)+g(x^2+y)$ where the functions f,g are arbitrary, prove that $\frac{\partial^2 z}{\partial x^2}-\frac{1}{x}\frac{\partial z}{\partial x}=4x^2\frac{\partial^2 z}{\partial y^2}$.

6. Find the Particular integral of the equation $(D^2-D^1)z=2y-x^2$.

7. Show that the surfaces $(x^2+y^2)^2-2a^2(x^2-y^2)+a^4=c$ can form a family of equipotential surfaces, and find the general form of the corresponding potential function.

8. Explain Dirichlet Problem.

[P.T.O]

SECTION – B

Answer FOUR of the following questions choosing ONE from each Unit.

(Marks : 4 × 15 marks = 60 marks)

UNIT – I

9. (a) (i) Find the integral curves of the equations $\frac{dx}{y(x+y)+az}=\frac{dy}{x(x+y)-az}=\frac{dz}{z(x+y)}$.

(ii) Find the orthogonal trajectories on the conicoid $(x+y)z=1$ of the conics in which it is cut by the system of planes $x-y+z=k$ where is k a parameter.

Or

(b) (i) Prove that A necessary and sufficient condition that the Pfaffian differential equation $X \cdot dr=0$ should be integrable is that $X \cdot curl\ X=0$.

(ii) Solve the equation $a^2 y^2 z^2\, dx+b^2 z^2 x^2\, dy+c^2 x^2 y^2\, dz=0$.

UNIT – II

10. (a) Show that the general solution of the linear partial differential equation $Pp+Qq=R$ is $F(u,v)=0$ where F is an arbitrary function and $u(x,y,z)=c_1$ and $v(x,y,z)=c_2$ form a solution of the equation $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$.

Or

(b) (i) Find the general integral of the partial differential equation $y^2p-xyq=x(z-2y)$.

(ii) Find the surface which intersects the surfaces of the system $z(x+y)=c(3z+1)$ Orthogonally and which passes through the circle $x^2+y^2=1$, $z=1$.

UNIT – III

11. (a) (i) Show that if f and g are arbitrary functions of a single variable, then $u=f(x-vt+i\alpha y)+g(x-vt-i\alpha y)$ is a solution of the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ provided that $\alpha^2=1-\frac{v^2}{c^2}$.

(ii) If the operator $F(D,D^1)$ is reducible, then prove that the order in which the linear factors occur is unimportant.

Or

(b) (i) Solve the equation $r-s+2q-z=x^2y^2$.

(ii) Reduce the equation $(n-1)^2\frac{\partial^2 z}{\partial x^2}-y^{2n}\frac{\partial^2 z}{\partial y^2}=ny^{2n-1}\frac{\partial z}{\partial y}$ to canonical form, and find its general solution.

UNIT – IV

12. (a) (i) Prove that the solutions of a certain Neumann problem can differ from one another by a constant only.

(ii) Prove that the solution $\psi(r, \theta, \phi)$ of the exterior dirichlet problem for the unit sphere $\nabla^2\psi = 0$, $r > 1$, $\psi = f(\theta, \phi)$ on $r = 1$ is given in terms of the solution $v(r, \theta, \phi)$ of the interior dirichlet problem $\nabla^2 v = 0$, $r < 1$, $v = f(\theta, \phi)$ on $r = 1$ by the formula $\psi(r, \theta, \phi) = \frac{1}{r} v\left(\frac{1}{r}, \theta, \phi\right)$.

Or

(b) A uniform insulated sphere of dielectric constant k and radius a carries on its surface a charge of density $\lambda P_n(\cos\theta)$. Prove that the interior of the sphere contributes an amount $\frac{8\pi^2 \lambda^2 a^2 kn}{(2n+1)(kn+n+1)^2}$ to the electrostatic energy.
