CBCS/ SEMESTER SYSTEM

(w.e.f. 2020-21 Admitted Batch)

B.A./B.Sc. MATHEMATICS

COURSE-V, LINEAR ALGEBRA

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any <u>FIVE</u> questions. Each question carries <u>FIVE</u> marks 5 X 5 M=25 M

1. Let p, q, r be fixed elements of a field F. Show that the set W of all triads (x, y, z) of elements of F, such that px+qy+rz=0 is a vector subspace of $V_3(R)$.

2. Define linearly independent &linearly dependent vectors in a vector space. If α, β, γ are linearly independent vectors of $V(R)$ then show that $\alpha+\beta, \beta+\gamma, \gamma+\alpha$ are also linearly independent.

3. Prove that every set of $(n+1)$ or more vectors in an n dimensional vector space is linearly dependent.

4. The mapping T : $V_3(R) \to V_3(R)$ is defined by T(x,y,z) = (x-y,x-z). Show that T is a linear transformation.

5. Let $T: R^3 \to R^2$ and $H: R^3 \to R^2$ be defined by T (x, y, z)= (3x, y+z) and H (x, y, z)= (2x-z, y). Compute i) T+H ii) 4T-5H iii) TH iv) HT.

6. If the matrix A is non-singular, show that the eigen values of A^{-1} are the reciprocals of the eigen values of A.

7. State and prove parallelogram law in an inner product space V(F).

8. Prove that the set S = $\left\{\left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right), \left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}\right)\right\}$ is an orthonormal set in the inner product space $R^3(R)$ with the standard inner product.

SECTION - B

Answer <u>ALL</u> the questions. Each question carries <u>TEN</u> marks. 5 X 10 M = 50 M

9(a)) Define vector space. Let V (F) be a vector space. Let W be a non empty sub set of V. Prove that the necessary and sufficient condition for W to be a subspace of V is $a, b \in F$ and $\alpha, \beta \in V \Rightarrow a\alpha + b\beta \in W$.

(OR)

(b) Prove that the four vectors (1,0,0), (0,1,0), (0,0,1) and (1,1,1) of $V_3(C)$ form linearly dependent set, but any three of them are linearly independent.

10(a)Define dimension of a finite dimensional vector space. If W is a subspace of a finite dimensional vector space V(F) then prove that W is finite dimensional and dim W$\leq n.$

(OR)

(b) If W be a subspace of a finite dimensional vector space V(F) then Prove that

$$\dim {}^{V}/_{W} = \dim V - \dim W.$$

11(a) Find T (x, y, z) where $T: R^3 \rightarrow R$ is defined by T (1, 1, 1) =3, T (0, 1, -2) =1, T (0, 0, 1) = -2

(OR)

(b) State and prove Rank Nullity theorem.

12(a) Find the eigen values and the corresponding eigen vectors of the matrix

$$A=\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}.$$

(OR)

(b) State and prove Cayley-Hamilton theorem.

13(a) State and prove Schwarz's inequality in an Inner product space V(F).

(OR)

(b) Given $\{(2,1,3),(1,2,3),(1,1,1)\}$ is a basis of R^3(R).Construct an orthonormal basis using Gram-Schmidt orthogonalisation process.