CBCS/ SEMESTER SYSTEM

(w.e.f. 2020-21 Admitted Batch)

B.A./B.Sc. MATHEMATICS

COURSE-IV, REAL ANALYSIS

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any <u>FIVE</u> questions. Each question carries <u>FIVE</u> marks 5 X 5 M=25 M

1.Prove that every convergent sequence is bounded.

2. Show that $\lim\left(\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2}\right.$+------------------+$\left.\frac{1}{(n+n)^2}\right)$=0.

3. Test the convergenceof the series $\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1} - n\right)$.

4. Examine for continuity of the function *f* defined by $f(x) = |x| + |x-1|$ at x=0 and 1.

5. Show that $f(x) = x\sin\frac{1}{x},\ x \neq 0; f(x) = 0, x = 0$ is continuous but not derivable at x=0.

6.Verify Rolle's theorem for the function f(x) = x^3- $6x^2$+11x-6 on $[1,3]$.

7. If $f(x) - x^2\ \forall\, x \in [0,1]$ and $p - \{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$ then find L(p, f) and U(p, f).

8.prove that if $f: [a,b] \to R$ is continuous on [a, b] then f is R- integrable on [a, b].

SECTION –B

Answer <u>ALL</u> the questions. Each question carries <u>TEN</u> marks. 5 X 10 M = 50 M

9.(a)If $s_n = 1 + \frac{1}{2!} + \frac{1}{3!} + - - - - - - - + \frac{1}{n!}$ then show that $\{s_n\}$ converges.

(OR)

(b) State and prove Cauchy's general principle of convergence.

10.(a) State and Prove Cauchy's nth root test.

(OR)

(b) Test the convergence of $\sum \frac{x^n}{x^n+a^n}$ $(x > 0, a > 0)$.

11.(a) Let f: R→ R be such that

$$f(x) = \frac{\sin(a+1)x+\sin x}{x} \quad \text{for } x < 0$$

$$= c \qquad \text{for } x = 0$$

$$= \frac{(x+bx^2)^{1/2}-x^{1/2}}{bx^{3/2}} \quad \text{for } x > 0$$

Determine the values of a, b, c for which the function f is continuous at x=0.

(OR)

(b) Define uniform continuity, If a function f is continuous on [a b] then f is uniformly continuous on [a b]

12.(a) Using Lagrange's theorem, show that $x > \log(1+x) > \frac{x}{(1+x)} \forall x > 0$.

(OR)

(b) State and prove Cauchy's mean value theorem.

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13.(a) State and prove Riemman's necessary and sufficient condition for R- integrability.

(OR)

(b) Prove that $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\cos x} dx \leq \frac{\pi^3}{6}$.