CBCS/ SEMESTER SYSTEM

(w.e.f. 2020-21 Admitted Batch)

B.A./B.Sc. MATHEMATICS

COURSE-III, ABSTRACT ALGEBRA

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks 5 X 5 M=25 M

1. Show that the set G=$\{x/x = 2^a 3^b \text{ and } a, b \in Z\}$ is a group under multiplication

2. Define order of an element. In a group G, prove that if $a \in G$ then $O(a) = O(a)^{-1}$.

3. If H and K are two subgroups of a group G, then prove that HK is a subgroup ⇔ HK=KH

4.If G is a group and H is a subgroup of index 2 in G then prove that H is a normal subgroup.

5. Examine whether the following permutations are even or odd

i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 1 & 4 & 3 & 2 & 5 & 7 & 8 & 9 \end{pmatrix}$

ii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 5 & 6 & 7 & 1 \end{pmatrix}$

6. Prove thata group of prime order is cyclic.

7.Prove that the characteristic of an integral domain is either prime or zero.

8. If F is a field then prove that $\{0\}$ and F are the only ideals of F.

SECTION - B

Answer ALL the questions. Each question carries TEN marks. 5 X 10 M = 50 M

9 a) Show that the set of n^{th} roots of unity forms an abelian group under multiplication.

(Or)

9 b) In a group G, for $a, b \in G$, O(a)=5, b ≠ e and $aba^{-1} = b^2$. Find O(b).

10 a) The Union of two subgroups is also a subgroup ⇔ one is contained in the other.

(Or)

b) State and prove Langrage’s theorem.

11 a) Prove that a subgroup H of a group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

(Or)

11 b) State and prove fundamental theorem of homomorphisms of groups.

12 a) Let S_n be the symmetric group on n symbols and let A_n be the group of even permutations. Then show that A_n is normal in S_n and $O(A_n) = \frac{1}{2}(n!)$

(Or)

12 b)prove thatevery subgroup of cyclic group is cyclic.

13 a) Prove that every finite integral domain is a field.

(Or)

13 b) Define principal idea. Prove that every ideal of Z is a principal ideal.