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M.Sc. DEGREE EXAMINATION, APRIL 2018

FOURTH SEMESTER

Branch – Mathematics

Paper V — MATHEMATICAL STATISTICS

Time : 3 Hours *Max. Marks : 80*

SECTION – A

Answer any FIVE of the following.

(Marks : 5 × 4 marks = 20 marks)

1. Two cards are drawn from an ordinary deck of 52 cards but the first card drawn is replaced before the second card is drawn. What is the probability that at least one of the cards will be a spade?

2. If a poker hand of 5 cards is drawn from a deck, what is the probability that it will contain 2 aces?

3. Use Chebychev's inequality to determine how many time a fair coin must be tossed in order that the probability will be at least 0.90 that the ration of the observed number of heads to the number of tosses will lie between 0.4 and 0.6.

4. An urn contains 7 white and 3 red balls. Two balls are drawn together, at random from this urn. Compute the probability that neither of them is white. Find also the probability of getting one white and one red ball. Hence compute the expected number of white balls drawn.

5. Find the moment generating function of binomial distribution.

6. If x is normally distributed with $\mu = 1$ and $\sigma = \frac{1}{2}$. Find $P\{0 < x < 1\}$.

7. Define type I and type II errors and maximum likelihood estimator.

8. Write the steps in solving testing of hypothesis problem.

SECTION – B

Answer ONE question from each Unit.

(Marks : 4 × 15 marks = 60 marks)

UNIT – I

9. (a) What are the axioms of probability? State and prove multiplicative theorem for probability.

   (b) Given the continuous density function $f(x) = c$, $1 < x < 3$ determine the value of c and calculate $p\{x < 2\}$.

Or

10. (a) State and prove Baye's formula.

(b) A speaks truth 4 out of 5 times. A die is tossed. He reports that there is a six. [...] the chance that actually there was six?

UNIT – II

11. (a) A random variable X has the following probability function :

Value of X, x :	-2	-1	0	1	2	3
$p(x)$:	0.1	k	0.2	$2k$	0.3	k

Find the value of k, and calculate mean and variance.

(b) Suppose that two-dimensional continuous random variable (X, Y) has joint p.d.f. by $f(x,y)=\begin{cases}6x^2y, & 0<x<1, 0<y<1\\ 0, & \text{elsewhere.}\end{cases}$

(i) Verify that $\int_0^1\int_0^1 f(x,y)\,dx\,dy = 1$

(ii) Find $P\left(0<X<\frac{3}{4}, \frac{1}{3}<Y<2\right)$, $P(X+Y<1)$, $P(X>Y)$ and $P(X<1 | Y<2)$.

Or

12. (a) Two random variables X and Y have the following joint probability density function

$$f(x,y)=\begin{cases}2-x-y, & 0\le x\le 1, 0\le y\le 1\\ 0, & \text{elsewhere.}\end{cases}$$

Find

(i) Marginal probability density function of X and Y;

(ii) Conditional density function.

(iii) Var (X) and Var (Y): and

(iv) Covariance between X and Y.

(b) State and prove weak law of large numbers.

UNIT – III

13. Fit a binomial function to the following data on the number of seeds germinating o[...] seeds on damp filter paper for 80 sets of seeds.

x:	0	1	2	3	4	5	6	7	8	9	10
f:	6	20	28	12	8	6	0	0	0	0	0

Or

14. A random sample of 10 boys had the following I.Q's : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q. of 100? Find a reasonable range in which most of the mean I.Q values of samples of 10 boys lie.

UNIT – IV

15. State and Prove CRAMER-RAO inequality.

Or

16. State and Prove Neyman — Pearson lemma.