(040940302/445931B)

M.Sc. DEGREE EXAMINATION, APRIL 2018

FOURTH SEMESTER

Branch — Mathematics

Paper III — GRAPH THEORY

Time : 3 Hours *Max. Marks : 80*

PART – A

Answer any FIVE of the following.

(Marks : 5 × 4 marks = 20 marks)

UNIT – I

1. Define Graph Isomorphism and show that there are eleven non isomorphic simple graphs on four vertices.

2. With usual notation, Show that $\sum_{\upsilon \in V} d(\upsilon) = 2\varepsilon$ and deduce that in any graph, the number of vertices of odd degree is even.

3. Show that if any two vertices of a loopless graph G are connected by a unique path, then G is a tree.

4. If e is a link of G, then show $\tau(G) = \tau(G-e) + \tau(G.e)$.

5. Show that if G is simple and 3-regular, then $k = k'$.

6. If G is 2-connected, then show that any two vertices of G lie on a common cycle.

7. If G is hamiltonian then show that for every non-empty proper subset S of V $\omega(G-S) \leq |S|$.

8. Show that if G is a simple graph with $\vartheta \geq 3$ and $\varepsilon > \binom{\vartheta-1}{2} + 1$ then G is hamiltonian.

PART – B

Answer ONE question from each Unit.

(Marks : 4 × 15 marks = 60 marks)

9. (a) Show that if G is simple and bipartite, then $\varepsilon \leq \frac{\vartheta^2}{4}$.

   (b) With usual notation, show that a $\omega(G) \leq \omega(G-e) \leq \omega(G)+1$, for $e \varepsilon E$.

Or

10. Prove that every properly labeled simplicial subdivision of a triangle has an odd number of distinguished triangles.

[P.T.O]

11. Let G be a graph with $v-1$ edges. Show that the following three statements are equivale

(a) G is connected;

(b) G is acyclic;

(c) G is a tree.

Or

12. Show that $(K_n) = n^{n-2}$.

13. Prove that a graph G with $V \geq 3$ is 2 - connected if and only if any two vertices of G connected by at least two internally-disjoint paths.

Or

14. Define $H_{m,n}$ is graph. Show that $H_{m,n}$ is m - connected.

15. If G is a simple graph with $V \geq 3$ and $\delta \geq \frac{9}{2}$ then show that 0 is hamiltonian.

Or

16. Prove that if G is Eulerian then any trail in G constructed by Fleury's algorithm is an E tour of G.
